Joint Entrance Exam | Mains-2019

Paper Code -

9th April 2019 | Morning

PHYSICS , CHMISTRY & MATHEMATICS

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with only Black Ball Point Pen provided in the examination hall.

2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions. The maximum marks are **360**.

5. There are three parts in the question paper A, B, C consisting of **Physics, Mathematics** and **Chemistry** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for correct response.

6. Candidate will be awarded marks as stated above in instruction No. 5 for correct response of each question. $\frac{1}{4}$ (one-fourth) marks of the total marks allotted to the questions (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

8. For writing particulars/marking responses on *Side-1* and *Side-2* of the Answer Sheet use ***only Black Ball Point Pen*** provided in the examination hall.

9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.

10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **four** pages (Page **20-23**) at the end of the booklet.

11. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***

12. The CODE for this Booklet is **B.** Make sure that the CODE printed on Side-2 of the Answer Sheet is same as that on this Booklet. Also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.

13. **Do not fold or make any stray mark on the Answer Sheet.**

Joint Entrance Exam/IITJEE-2019

PART-A	PHYSICS

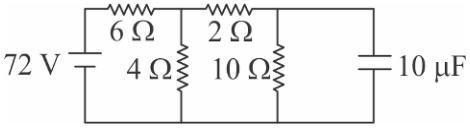
1. A concave mirror face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is:

(1) 0.16 m **(2)** 0.24 m **(3)** 0.32 m **(4)** 1.60 m

2. The magnetic field of a plane electromagnetic wave is given by: $\vec{B} = B_0\hat{i} - [\cos(kz - \omega t)] + B_1\hat{j}\cos(kz + \omega t)$ where $B_0 = 3\times10^{-5}T$ and $B_1 = 2\times10^{-6}T$. The rms value of the force experienced by a stationary charge $Q = 10^{-4}C$ at $z = 0$ is closed to:

(1) 0.1 N **(2)** 3×10^{-2} N **(3)** 0.6 N **(4)** 0.9 N

3. Determine the charge on the capacitor in the following circuit :



(1) 200 μC **(2)** 60 μC **(3)** 2 μC **(4)** 10 μC

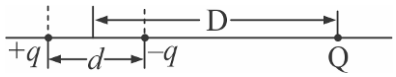
4. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3\sin(0.157x)\cos(200\pi t)$. The length of the string is: (All quantities are in SI units.)

(1) 60 m **(2)** 20 m **(3)** 80 m **(4)** 40 m

5. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(1) 1.0 kg **(2)** 1.5 kg **(3)** 1.8 kg **(4)** 1.2 kg

6. A system of three charges are placed as shown in the figure :



If $D >> d$, the potential energy of the system

(1) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{2D^2}\right]$ **(2)** $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{2qQd}{D^2}\right]$

(3) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$ **(4)** $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d} + \frac{qQd}{D^2}\right]$

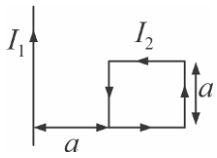
7. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be:

(1) 80 m/s **(2)** 100 m/s **(3)** $100\sqrt{5}$ m/s **(4)** $80\sqrt{5}$ m/s

8. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is:

(1) 4 : 3 : 2 **(2)** 14 : 15 : 20 **(3)** 10 : 15 : 17 **(4)** 2 : 3 : 4

9. A rigid square loop of side `a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be:



(1) Zero

(2) Attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$

(3) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

(4) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$

10. A signal $A\cos\omega t$ is transmitted using $v_0 \sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is:

(1) $v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$

(2) $(v_0 + A)\cos\omega t \sin\omega_0 t$

(3) $v_0 \sin[\omega_0(1 + 0.01\, A \sin\omega t)t]$

(4) $v_0 \sin\omega_0 t + A\cos\omega t$

11. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?

(1) 90° **(2)** 60° **(3)** 120° **(4)** 150°

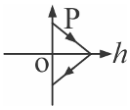
12. A uniform cable of mass `$M$' and length `L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be:

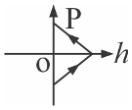
(1) $\frac{MgL}{2n^2}$ **(2)** $\frac{MgL}{n^2}$ **(3)** $nMgL$ **(4)** $\frac{2MgL}{n^2}$

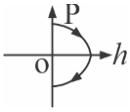
13. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\vec{v}, m$ is its mass and k_B is Bolzmann constant, then its temperature will be:

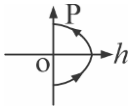
(1) $\frac{m\vec{v}^2}{5k_B}$ **(2)** $\frac{m\vec{v}^2}{3k_B}$ **(3)** $\frac{m\vec{v}^2}{7k_B}$ **(4)** $\frac{m\vec{v}^2}{6k_B}$

14. A ball is thrown vertically up (taken as + z-axis) from the ground. The correct momentum-height (p-b) diagram is:

(1) 

(2) 

(3) 

(4) 

15. A moving coil galvanometer has resistance 50Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a $5\,k\Omega$ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to:

(1) 10 V **(2)** 40 V **(3)** 15 V **(4)** 20 V

16. A capacitor with capacitance $5\,\mu F$ is charged to $5\,\mu C$. If the plates are pulled apart to reduce the capacitance to $2\,\mu F$, how much work is done?

(1) $2.55\times10^{-6}\,J$ **(2)** $3.75\times10^{-6}\,J$ **(3)** $6.25\times10^{-6}\,J$ **(4)** $2.16\times10^{-6}\,J$

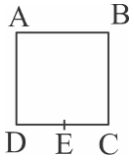
17. The pressure wave, $P=0.01\sin[1000t-3x]\,Nm^{-2}$, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day when temperature is *T*, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of *T* is:

(1) 12°C **(2)** 15°C **(3)** 11°C **(4)** 4°C

18. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is *I* then the angular acceleration of the disc is:

(1) $\frac{k}{2I}\theta$ **(2)** $\frac{k}{I}\theta$ **(3)** $\frac{k}{4I}\theta$ **(4)** $\frac{2k}{I}\theta$

19. A wire of resistance *R* is bent to form a square *ABCD* as shown in the figure. The effective resistance between *E* and *C* is: (*E* is mid-point of arm *CD*)



(1) $\frac{1}{16}R$ **(2)** R **(3)** $\frac{7}{64}R$ **(4)** $\frac{3}{4}R$

20. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be:

(1) 488.9 nm **(2)** 642.7 nm **(3)** 388.9 nm **(4)** 889.2 nm

21. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '$2r$' is:

(1) M **(2)** $\frac{M}{2}$ **(3)** $2M$ **(4)** $4M$

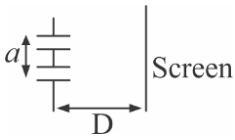
22. A solid sphere mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness $2a$ and mass $2M$. The gravitational field at distance '$3a$' from the centre will be:

(1) $\frac{GM}{9a^2}$ **(2)** $\frac{2GM}{9a^2}$ **(3)** $\frac{GM}{3a^2}$ **(4)** $\frac{2GM}{3a^2}$

23. A rectangular coil (Dimension $5\,cm\times2.5\,cm$) with 100 turns, carrying a current of $3A$ in the clock-wise direction, is kept centered at the origin and in the *X*-*Z* plane. A magnetic field of 1 *T* is applied along *X*-axis. If the coil is tiled through 45° about *Z*-axis, then the torque on the coil is:

(1) 0.27 Nm **(2)** 0.38 Nm **(3)** 0.55 Nm **(4)** 0.42 Nm

24. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness *t* and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to *n* fringe widths. If the wavelength of light used is λ, t will be:



(1) $\frac{nD\lambda}{a(\mu-1)}$ **(2)** $\frac{2D\lambda}{a(\mu-1)}$ **(3)** $\frac{D\lambda}{a(\mu-1)}$ **(4)** $\frac{2nD\lambda}{a(\mu-1)}$

25. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is:

(1) $2T\sqrt{\frac{1}{10}}$ **(2)** $4T\sqrt{\frac{1}{15}}$ **(3)** $4T\sqrt{\frac{1}{14}}$ **(4)** $2T\sqrt{\frac{1}{14}}$

26. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to:

(1) L^2 **(2)** $1/L^2$ **(3)** L **(4)** $1/L$

27. An NPN transistor is used in common emitter configuration as an amplifier with $1\,k\Omega$ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and $15\,\mu A$ change in the base current of the amplifier. The input resistance and voltage gain are:

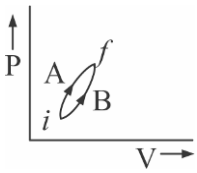
(1) $0.67\,k\Omega$, 200 **(2)** $0.33\,k\Omega$, 300 **(3)** $0.67\,k\Omega$, 300 **(4)** $0.33\,k\Omega$, 1.5

28. The electric field of light wave is given as $\vec{E}=10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}}-2\pi\times6\times10^{14}t\right)\hat{x}\frac{N}{C}$. This light falls on a metal plate of work function 2 eV. The stopping potential of the photo-electrons is:

Given, E (in eV) $=\frac{12375}{\lambda(\text{in Å})}$

(1) 2.0 V **(2)** 0.72 V **(3)** 0.48 V **(4)** 2.48 V

29. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then:



(1) $\Delta Q_A > \Delta Q_B, \Delta U_A = \Delta U_B$ **(2)** $\Delta Q_A < \Delta Q_B, \Delta U_A < \Delta U_B$

(3) $\Delta Q_A = \Delta Q_B, \Delta U_A = \Delta U_B$ **(4)** $\Delta Q_A > \Delta Q_B, \Delta U_A > \Delta U_B$

30. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is:

(1) 0.01 kg/m^3 **(2)** 0.10 kg/m^3 **(3)** 0.07 mg/m^3 **(4)** 0.31 kg/m^3

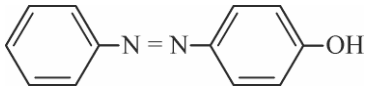
PART-B	CHEMISTRY

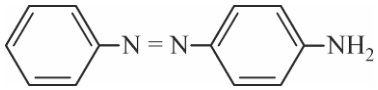
31. For reaction, $N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$; identify dihydrogen (H_2) as a limiting reagent in the following reaction mixtures.

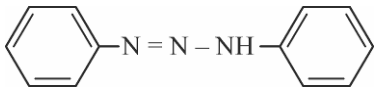
(1) 28g of N_2 + 6g of H_2 **(2)** 56g of N_2 + 10g of H_2

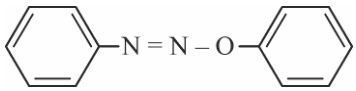
(3) 14g of N_2 + 4g H_2 **(4)** 35g of N_2 + 8g of H_2

32. Aniline dissolved in dilute HCL is reacted with sodium nitrate at 0°C. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is :

(1) 

(2) 

(3) 

(4) 

33. Among the following, the set of parameters that represents path functions, is:

(a) q + w (b) q (c) w (d) H – TS

(1) (b), (c) and (d) **(2)** (a), (b) and (c) **(3)** (b) and (c) **(4)** (a) and (d)

34. The aerosol is a kind of colloid in which :

(1) liquid is dispersed in water **(2)** gas is dispersed in liquid

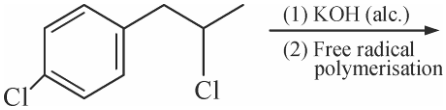
(3) gas is dispersed in solid **(4)** solid is dispersed in gas

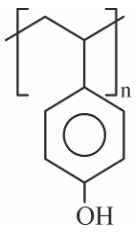
35. Magnesium powder burns in air to give :

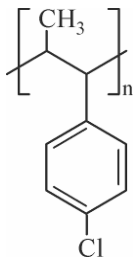
(1) MgO and Mg_3N_2 **(2)** MgO only

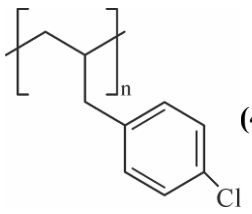
(3) MgO and $Mg(NO_3)_2$ **(4)** $Mg(NO_3)_2$ and wMg_3N_2

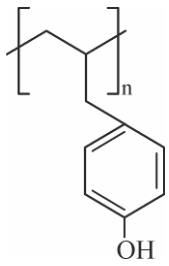
36. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

37. Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statement is:

(x_M = Mole fraction of 'M' in solutions ; x_N = Mole fraction of 'N' in solution ;

y_M = Mole fraction of 'M' in vapour phase ; y_N = Mole fraction of 'n' in vapour phase)

(1) $\frac{x_M}{x_N} = \frac{y_M}{y_N}$ **(2)** $\frac{x_M}{x_N} < \frac{y_M}{y_N}$

(3) $(x_M - y_M) < (x_N - y_N)$ **(4)** $\frac{x_M}{x_N} > \frac{y_M}{y_N}$

38. The number of water molecule(s) not coordinated to copper ion directly in $CuSO_4 \cdot 5H_2O$, is :

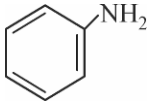
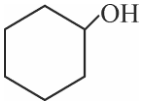
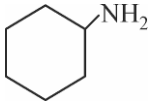
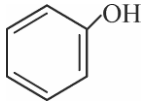
(1) 1 **(2)** 2 **(3)** 4 **(4)** 3

39. For any given series of spectral lines of atomic hydrogen, let $\Delta\vec{v} = \vec{v}_{max} - \vec{v}_{min}$ be the difference in maximum and minimum frequencies in cm^{-1}. The ratio $\Delta\vec{v}_{Lyman} / \Delta\vec{v}_{Balmer}$ is:

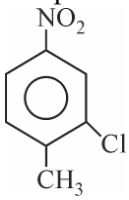
(1) 27 : 5 **(2)** 4 : 1 **(3)** 5 : 4 **(4)** 9 : 4

40. The organic compound that gives following qualitative analysis is :

	Test	**Inference**
(a)	Dil. HCl	Insoluble
(b)	NaOH solution	soluble
(c)	Br_2 / water	Decolourization

(1)  **(2)**  **(3)**  **(4)** 

41. The correct IUPAC name of the following compound is:



(1) 5-chloro-4-methyl-1-nitrobenze **(2)** 2-methy 1-5-nitro-1-chlorobenzene

(3) 3-chloro-4-methyl-1-nitrobenzene **(4)** 2-chloro-1-methyl-4-nitrobenzene

42. Excessive release of CO_2 into the atmosphere result in :

(1) Global warming **(2)** Formation of smog

(3) Polar vortex **(4)** Depletion of ozone

43. Which of the following statements is not true about sucrose?

(1) It is also named as invert sugar

(2) On hydrolysis, it produces glucose and fructose

(3) It is a non reducing sugar

(4) The glcosidic linkage is present between C_1 and β-frutose

44. The major product of the following reaction is :

$CH_3CH=CHCO_2CH_3 \xrightarrow{LiAlH_4}$

(1) $CH_3CH_2CH_2CHO$ **(2)** $CH_3CH_2CH_2CO_2CH_3$

(3) $CH_3CH_2CH_2CH_2OH$ **(4)** $CH_3CH=CHCH_2OH$

45. The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are:

(1) d_{z^2} and d_{xz} **(2)** d_{yz} and d_{z^2} **(3)** $d_{x^2-y^2}$ and d_{xy} **(4)** d_{xz} and d_{yz}

46. The major product of the following reaction is:

$CH_3C\equiv CH \xrightarrow[(ii)\,DI]{(i)\,DCl\ (1equiv.)}$

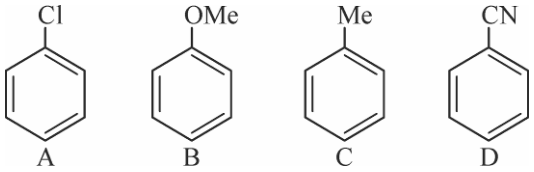
(1) $CH_3CH(Cl)CHD(l)$ **(2)** $CH_3C(I)(Cl)CHD_2$

(3) $CH_3CD_2CH(Cl)(I)$ **(4)** $CH_3CD(I)CHD(Cl)$

47. The element having greatest difference between its first and second ionization energies, is:

(1) Sc **(2)** Ca **(3)** Ba **(4)** K

48. The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is:



(1) $A < B < C < D$ **(2)** $B < C < A < D$ **(3)** $D < A < C < B$ **(4)** $D < B < A < C$

49. The ore that contains the metals in the form of fluoride is :

(1) magnetite **(2)** malachite **(3)** sphalerite **(4)** cryolite

50. The standard Gibbs energy for the given cell reaction is $kJ\,mol^{-1}$ at 298 K is :

$Zn(s) + Cu^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Cu(s)$

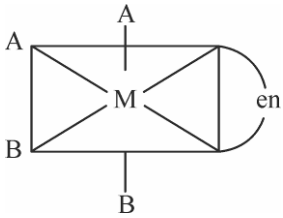
$E° = 2V$ at 298 K

(Faraday's constant, F = 96000 C mol^{-1})

(1) –192 **(2)** –384 **(3)** 192 **(4)** 384

51. The one that will show optical activity is:
(en = ethane – 1,2-diamine)

(1)



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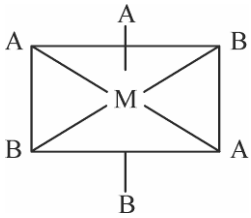

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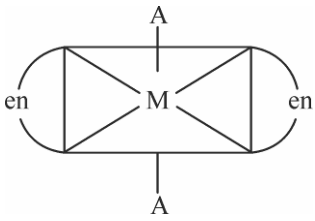

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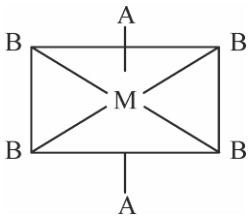


52. C_{60}, an allotrope of carbon contains :

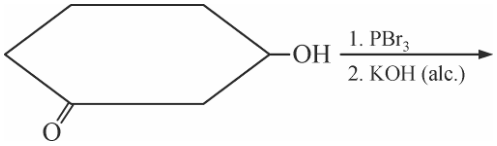
(1) 12 hexagons and 20 pentagons **(2)** 16 hexagons and 16 pentagons
(3) 20 hexagons and 12 pentagons **(4)** 18 hexagons and 14 pentagons

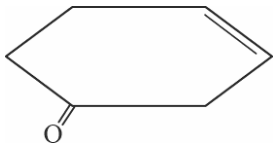
53. Match the catalysts (column I) with products (Column II)

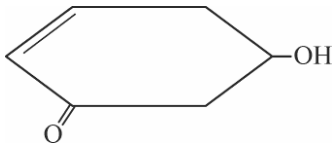
Column I (Catalyst)		Column II (Product)	
(A)	V_2O_5	**(i)**	Polyethylene
(B)	$TiCl_4/Al(Me)_3$	**(ii)**	Ethanal
(C)	$PdCl_2$	**(iii)**	H_2SO_4
(D)	Iron Oxide	**(iv)**	NH_3

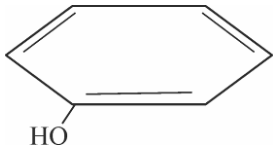
(1) (A)-(iii) ; (B)-(iv) ; (C)-(i) ; (D)-(ii) **(2)** (A)-(iv) ; (B)-(iii) ; (C)-(ii) ; (D)-(i)
(3) (A)-(iii) ; (B)-(i) ; (C)-(ii); (D)-(iv) **(4)** (A)-(ii) ; (B)-(iii) ; (C)-(i) ; (D)-(iv)

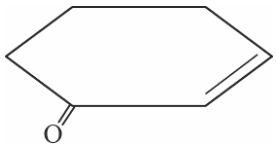
54. The major product of the following reaction is:



(1) 

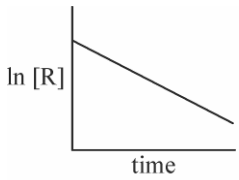
(2) 

(3) 

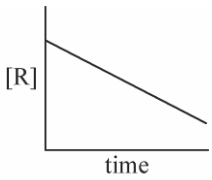
(4) 

55. The given plots represent the variation of the concentration of a reactant *R* with time for different reactions (i) (ii). The respective orders of the reactions are :

(1) 1, 0
(2) 0, 2
(3) 1, 1
(4) 0, 1



(i)



(ii)

56. The osmotic pressure of a dilute solution of an compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in $mol\,L^{-1}$) in solution is:

(1) 4×10^{-4} **(2)** 4×10^{-2} **(3)** 6×10^{-2} **(4)** 16×10^{-4}

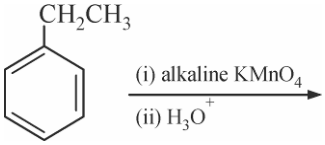
57. Consider the van der Waals constants, a and b, for the following gases.

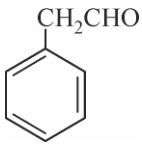
Gas	**Ar**	**Ne**	**Kr**	**Xe**
a/(atm $dm^6\,mol^{-2}$)	1.3	0.2	5.1	4.1
b/($10^{-2}\,dm^3 mol^{-1}$)	3.2	1.7	1.0	5.0

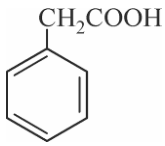
Which gas is expected to have the highest critical temperature?

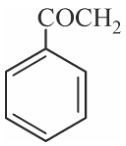
(1) Ar **(2)** Ne **(3)** Kr **(4)** Xe

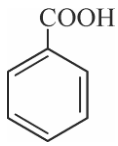
58. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

59. The correct order of the oxidation states of nitrogen in NO, N_2O, NO_2 and N_2O_3 is:

(1) $NO_2 < NO < N_2O_3 < N_2O$ **(2)** $N_2O < N_2O_3 < NO < NO_2$
(3) $N_2O < NO < N_2O_3 < NO_2$ **(4)** $NO_2 < N_2O_3 < NO < N_2O$

60. Among the following, the molecule expected to be stabilized by anion formation is : C_2, O_2, NO, F_2

(1) C_2 **(2)** NO **(3)** O_2 **(4)** F_2

PART-C	MATHEMATICS

61. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points $(1, f(1))$ and $(-1, f(-1))$, then S is equal to :

(1) $\left\{-\frac{1}{3}, 1\right\}$ **(2)** $\left\{-\frac{1}{3}, -1\right\}$ **(3)** $\left\{\frac{1}{3}, 1\right\}$ **(4)** $\left\{\frac{1}{3}, -1\right\}$

62. If the standard deviation of the numbers $-1, 0, 1, k$ is $\sqrt{5}$ where $k > 0$, then k is equal to :

(1) $\sqrt{6}$ **(2)** $2\sqrt{6}$ **(3)** $4\sqrt{\frac{5}{3}}$ **(4)** $2\sqrt{\frac{10}{3}}$

63. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is :

(1) $\frac{\pi - 2}{8}$ **(2)** $\frac{\pi - 2}{4}$ **(3)** $\frac{\pi - 1}{4}$ **(4)** $\frac{\pi - 1}{2}$

64. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is :

(1) 3/4 **(2)** $\frac{3}{2}(1 + \cos 20°)$ **(3)** $\frac{3}{4} + \cos 20°$ **(4)** 3/2

65. Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is :

(1) $\frac{5\pi}{3}$ **(2)** 2π **(3)** π **(4)** $\frac{13\pi}{6}$

66. If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is :

(1) 9 / 2 **(2)** $\sqrt{5}$ / 2 **(3)** $2\sqrt{5}$ **(4)** 7 / 2

67. If the function $f : R - \{1, -1\} \to$ A defined by $f(x) = \frac{x^2}{1 - x^2}$, is surjective, then A is equal to :

(1) $R - (-1, 0)$ **(2)** $R - [-1, 0)$ **(3)** $R - \{-1\}$ **(4)** $[0, \infty)$

68. If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdot \ldots\ldots\ldots \cdot \begin{bmatrix} 1 & n-1 \\ 0 & n \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$, then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is :

(1) $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$ **(2)** $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$ **(3)** $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$ **(4)** $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$

69. If $f(x)$ is a non-zero polynomial of degree four, having local extreme points at $x = -1, 0, 1$; then the set $S = \{x \in R : f(x) = f(0)\}$ contains exactly :

(1) four rational numbers **(2)** two irrational and two rational numbers
(3) two irrational and one rational number **(4)** four irrational numbers

70. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is :

(1) $\frac{2}{\sqrt{5}}$ **(2)** $\frac{\sqrt{5}}{2}$ **(3)** $\frac{\sqrt{15}}{2}$ **(4)** $\frac{3}{\sqrt{5}}$

71. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane $y - z + 5 = 0,$ also, passes through the point :

(1) $(-\sqrt{2}, 1, -4)$ **(2)** $(\sqrt{2}, 1, 4)$ **(3)** $(\sqrt{2}, -1, 4)$ **(4)** $(-\sqrt{2}, -1, -4)$

72. Let $\hat{\alpha} = 3\hat{i} + \hat{j}$ and $\hat{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2,$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha},$ then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to :

(1) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ **(2)** $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$

(3) $\frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$ **(4)** $3\hat{i} - 9\hat{j} - 5\hat{k}$

73. Let the sum of the first *n* terms of a non-constant A.P., $a_1, a_2, a_3, \ldots\ldots$ be $50n + \frac{n(n-7)}{2}A,$ where *A* is a constant. If *d* is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to :

(1) $(50, 50 + 46A)$ **(2)** $(A, 50 + 45A)$ **(3)** $(A, 50 + 46A)$ **(4)** $(50, 50 + 45A)$

74. If the function *f* defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by $f(x) = \begin{cases} \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, & x \neq \frac{\pi}{4} \\ k, & x = \frac{\pi}{4} \end{cases}$ is continuous, then *k* is equal to :

(1) $\frac{1}{\sqrt{2}}$ **(2)** 1 **(3)** 2 **(4)** $\frac{1}{2}$

75. All the points in the set $S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\} (i = \sqrt{-1})$ lie on a :

(1) straight line whose slope is 1 **(2)** circle whose radius is 1

(3) straight line whose slope is –1 **(4)** circle whose radius is $\sqrt{2}$

76. If the fourth term in the Binomial expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6 (x > 0)$ is $20 \times 8^7,$ then a value of *x* is :

(1) 8^{-2} **(2)** 8 **(3)** 8^2 **(4)** 8^3

77. Let $f(x) = 15 - |x - 10|;\ x \in R.$ Then the set of all values of *x*, at which the function, $g(x) = f(f(x))$ is not differentiable, is :

(1) {10} **(2)** {5, 10, 15} **(3)** {5, 10, 15, 20} **(4)** {10, 15}

78. Let $p, q \in R.$ If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0,$ then :

(1) $q^2 - 4p - 16 = 0$ **(2)** $p^2 - 4q + 12 = 0$

(3) $p^2-4q-12=0$ **(4)** $q^2+4p+14=0$

79. The integral $\int \sec^{2/3}x\,\mathrm{cosec}^{4/3}x\,dx$ is equal to :

(Here C is a constant of integration)

(1) $3\tan^{-1/3}x+C$ **(2)** $-3\cot^{-1/3}x+C$

(3) $-\frac{3}{4}\tan^{-4/3}x+C$ **(4)** $-3\tan^{-1/3}x+C$

80. If the tangent to the curve, $y=x^3+ax-b$ at the point (1, –5) is perpendicular to the line, $-x+y+4=0,$ then which one of the following points lies on the curve ?

(1) (–2, 1) **(2)** (–2, 2) **(3)** (2, –1) **(4)** (2, –2)

81. If a tangent to the circle $x^2+y^2=1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is :

(1) $x^2+y^2-2xy=0$ **(2)** $x^2+y^2-16x^2y^2=0$

(3) $x^2+y^2-2x^2y^2=0$ **(4)** $x^2+y^2-4x^2y^2=0$

82. Four persons can hit a target correctly with probabilities $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is :

(1) $\frac{7}{32}$ **(2)** $\frac{25}{192}$ **(3)** $\frac{25}{32}$ **(4)** $\frac{1}{192}$

83. The area (in sq. units) of the region $A=\{(x,\,y):x^2\le y\le x+2\}$ is :

(1) $\frac{13}{6}$ **(2)** $\frac{9}{2}$ **(3)** $\frac{10}{3}$ **(4)** $\frac{31}{6}$

84. Let $\sum_{k=1}^{10} f(a+k)=16(2^{10}-1),$ where the function f satisfies $f(x+y)=f(x)f(y)$ for all natural numbers x, y and $f(1)=2.$ Then the natural number 'a' is :

(1) 16 **(2)** 3 **(3)** 4 **(4)** 2

85. Let α and β be the roots of the equation $x^2+x+1=0.$ Then for $y\ne 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to:

(1) $y(y^2-3)$ **(2)** y^3 **(3)** $y(y^2-1)$ **(4)** y^3-1

86. The solution of the differential equation $x\frac{dy}{dx}+2y=x^2(x\ne 0)$ with $y(1)=1,$ is :

(1) $y=\frac{4}{5}x^3+\frac{1}{5x^2}$ **(2)** $y=\frac{x^3}{5}+\frac{1}{5x^2}$

(3) $y=\frac{x^2}{4}+\frac{3}{4x^2}$ **(4)** $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

87. If one end of a focal chord of the parabola, $y^2=16x$ is at (1, 4), then the length of this focal chord is :

(1) 20 **(2)** 25 **(3)** 22 **(4)** 24

88. Slope of a line passing through $P(2, 3)$ and intersecting the line, $x+y=7$ at a distance of 4 units from P, is :

(1) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ **(2)** $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ **(3)** $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ **(4)** $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

89. For any two statements p and q, the negation of the expression $p\vee(\sim p\wedge q)$ is :

(1) $\sim p\vee\sim q$ **(2)** $p\wedge q$ **(3)** $\sim p\wedge\sim q$ **(4)** $p\leftrightarrow q$

90. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with a least 6 males and n is the number of ways the committee is formed with at least 3 females, then :

(1) $m=n=68$ **(2)** $n=m-8$ **(3)** $m=n=78$ **(4)** $m+n=68$